

# Adding kinetic effects to finite difference frequency domain simulation of ion cyclotron heating\*



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## Introduction and Summary

The kinetic plasma current required for linear full-wave simulation of radio-frequency (RF) heating of Tokamak plasmas is typically included in frequency-domain simulation by application of a spectral method<sup>2</sup> and the Stix hot plasma dielectric tensor<sup>1</sup>. This dielectric constrains the choice of basis when applying the spectral method to the Fourier set which in turn creates difficulties when simulating bounded domains with complex geometries, limits the spatial resolution to be uniform, and requires the factorization of a large, dense linear system whose solution time scales unfavorably as  $N^3$ . To overcome these issues we propose an iterative method for including the kinetic physics via direct integration of the RF force on a discrete set of phase space trajectories under a given estimate of the RF electric wave field. Here we present an initial comparison of the parallel kinetic plasma response of the two methods.

## Motivation, Method and Initial Results

Injection of RF waves in Tokamak plasmas is an invaluable tool for heating and current profile control. Due to the difficulty in directly measuring RF wave propagation in plasmas, heating and current drive scenarios are typically developed and studied using computer simulation on leadership class supercomputers (e.g., ORNL's Jaguar). Typical frequency-domain (FD) kinetic simulations in the ion-cyclotron and lower-hybrid frequency ranges rely on Fourier space expressions for the linear plasma response and requires the solution of a large, dense matrix ( $N^3$  operations). Furthermore, the Fourier basis is ill-suited to bounded domains and variable resolution grids. With a goal of coupling a high fidelity antenna model to a hot plasma, these issues are significant. Our goal for this work is to develop an approach to including kinetic effects in finite-difference FD simulation that scales better than  $N^3$ , can efficiently exploit upcoming supercomputing architectures (GPUs), include more physics (e.g., the plasma sheath), and be employed as a kinetic extension to existing finite-difference / finite-element packages.



**The traditional Stix approach** derives analytic expressions<sup>1</sup> for the kinetic plasma response under the electric field of each Fourier mode assuming unperturbed, straight particle trajectories in an unbounded plasma. Here we limit ourselves to the problem of parallel propagation ( $k_{\perp}=0$ ,  $k_{\parallel}=22.1 \text{ m}^{-1}$ ) in a thermal electron plasma with  $n_e=1.1 \times 10^{14} \text{ m}^{-3}$  such that the Stix parallel plasma current is given by

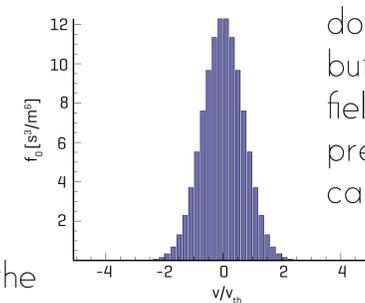
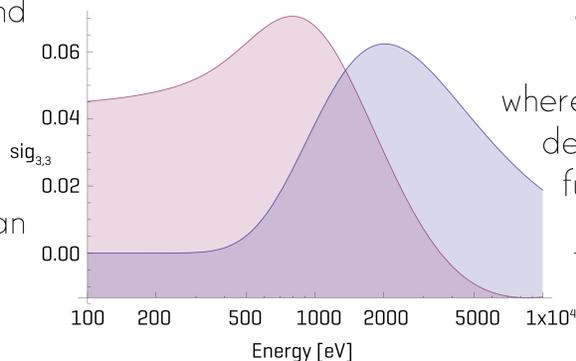
$$j_{\parallel} = \sigma_{3,3} E_{\parallel}$$

with the parallel conductivity and its cold plasma limit being

$$\sigma_{3,3} = \frac{i\epsilon_0\omega_{pe}^2}{k_{\parallel}v_{th}} \zeta Z'(\zeta) \rightarrow \frac{i\epsilon_0\omega_{pe}^2}{\omega}$$

where  $Z'(\zeta) = -2(1 + \zeta Z(\zeta))$  is the derivative of the plasma dispersion function with argument  $\zeta = \frac{\omega}{k_{\parallel}v_{th}}$ .

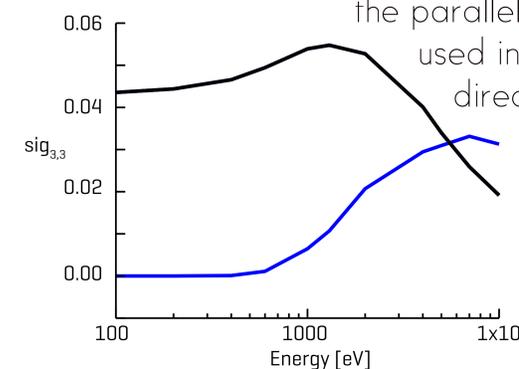
The figure left shows the real and imaginary parts of  $\text{sig}_{3,3}$  for a single parallel mode.



**A numerical approach** to calculating the FD plasma response does not require a decomposition of electric field into modes, but rather may start with any estimate of the wave electric field and iteratively add kinetic effects. This is similar to previous lower-hybrid work of Shiraiwa et al.<sup>3</sup> except we calculate the kinetic response as an integral over a discrete

$$\mathbf{j}_p(\mathbf{r}, t) = \frac{q}{m} \int \mathbf{v} \int_t^{-\infty} w(t') \mathbf{E}(\mathbf{r}', t') dt' f_0(\mathbf{v}) d\mathbf{v}$$

set of phase space trajectories ( $\mathbf{r}', t'$ ). The  $w(t')$  is a temporal weighting function to ensure the finite time integral extent is not important. Here we attempt a proof-of-principle by calculating the parallel plasma current for the single mode used in the center panel. The image here is directly comparable to that in the center panel, although not identical (yet). We expect that further work will see these responses match. It is unclear at this point how fast iterative application of this approach will converge to the correct kinetic response, if at all.



[1] T. H. Stix, 'Waves in Plasmas', AIP Press ISBN 0-88318-859-7 [1992]

[2] E. A. Coutsias et al., Phys. Scr., 40, 270 [1989]

[3] S. Shiraiwa et al., Phys. Plasmas, 17, 056119 [2010]